Lecture 12
CSR wake and its microwave instability

January 24, 2019

Lecture outline

- Radiation in free space and radiation reaction force
- CSR wakefield in free space
- Shielding and CSR wakefield with shielding
- Haïssinski equilibrium with CSR wake
- Microwave instability driven by CSR wake

CSR wake

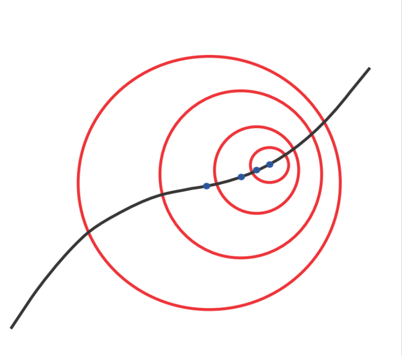
When a bunch of charged particles emits radiation, the energy of the electromagnetic field is taken from its kinetic energy. The energy balance in the process is maintained through a force that acts in the direction opposite to the velocity of the bunch. This force is called the *radiation reaction force*. In accelerator physics it is also known as the *coherent synchrotron radiation wake*, or *CSR wake*. In this lecture we discuss this wake and the associated microwave instability that it drives in relativistic beams.

The CSR wake in free space differs from the classical wakes that we considered previously. It is generated when the beam is moving along a curvilinear orbit. It is localized in front of the source particle, rather than behind it.

The CSR wake is important in modern accelerator with relatively short bunches and high peak currents, such as free electron lasers. The CSR instability has been observed in many accelerator based circular light sources.

How a moving charge emits electromagnetic waves

A constantly accelerating charge is radiating spherical shells of EM field at each moment of time, and they expand increasing their radii with the speed of light[36]. Each sphere has its center at the point where it was emitted. The corresponding time is called the *retarded* time and the position of the particle at the retarded time is the *retarded* position.



If a particle's orbit is given by the vector-function $\boldsymbol{r}_0(t)$, and we observe a sphere at time t at point $\boldsymbol{r}$ in space, then the retarded time t_{ret} is determined from the equation

$$c(t - t_{\mathrm{ret}}) = |\boldsymbol{r} - \boldsymbol{r}_0(t_{\mathrm{ret}})| \qquad (12.1)$$

and the retarded position is $\boldsymbol{r}_0(t_{\mathrm{ret}})$. Eq. (12.1) has always one solution for t_{ret} with $t_{\mathrm{ret}} < t$.

[36] More details on this description of radiation can be found in the textbook: G. Stupakov and G. Penn. *Classical Mechanics and Electromagnetism in Accelerator Physics*, Springer, 2018.

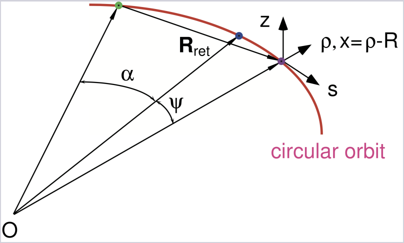
CSR wake in free space

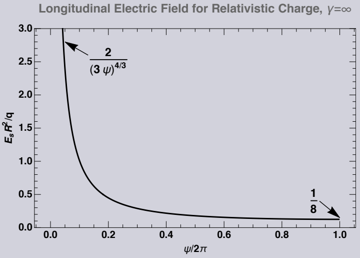
In the simplest model, the CSR wake field is derived in free space (no metal boundaries around). A line-charge beam is assumed with all particles traveling along a *circular orbit* of radius ρ with $v = c$. This model is applicable for long bending magnets whose length is much larger than the formation length of the radiation. We are interested in bunch length $\sigma_z \ll \rho$, where ρ is the bending radius. As it turns out the wake is localized in front of the source charge, rather than behind, as in classical wakes[37].

[37] More precisely, the wake behind is much smaller than in front of the charge.

CSR wake in free space

The wake can be calculated with the help of Lienard-Wiehert fields as the field of a relativistic charge, $v = c$, moving on a circular orbit of radius ρ ($R \to \rho$).





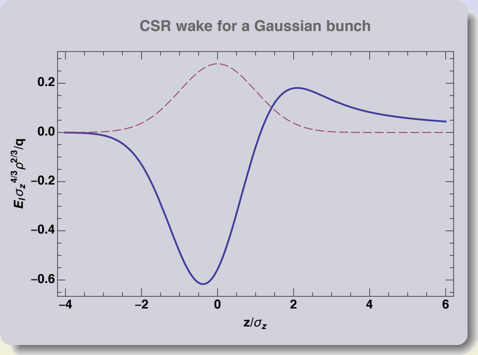
The longitudinal electric field of the bunch $E_\ell(s)$ is given by the following integral

$$E_\ell(z) = -\frac{1}{4\pi\epsilon_0}\frac{2eN}{3^{1/3}\rho^{2/3}}\int_{-\infty}^{z}\frac{1}{(z-z')^{1/3}}\frac{\partial\lambda(z')}{\partial z'}\,dz' , \qquad (12.2)$$

where N is the number of particles in the bunch. Note that the integration assumes $z' < z$ which corresponds to the wake localized in front of the particle. [The energy loss per unit length is, of course, eE_ℓ.]

CSR wake in a Gaussian bunch

For a Gaussian distribution, $\lambda(z) = (2\pi)^{-1/2}\sigma_z^{-1} e^{-z^2/2\sigma_z^2}$, the last integral can be computed numerically.



The distance is measured in units of σ_s, and the field is measured in units of $Q/\sigma_z^{4/3}\rho^{2/3}$, where Q is the total charge of the bunch.

CSR wake and impedance

To find the CSR wake of a point charge we integrate by parts Eq. (12.2)

$$E_\ell(z) = \frac{1}{4\pi\epsilon_0}\frac{2eN}{\rho^{2/3}3^{4/3}}\int_{-\infty}^{z}\frac{1}{(z-z')^{4/3}}\lambda(z')\,dz' \qquad (12.3)$$

This means that the CSR wake *per unit length* of a point charge $(\lambda(z) \to \delta(z))$ is

$$w_{csr}(s) = -\frac{Z_0 c}{4\pi}\frac{2h(-s)}{3^{4/3}\rho^{2/3}(-s)^{4/3}}$$

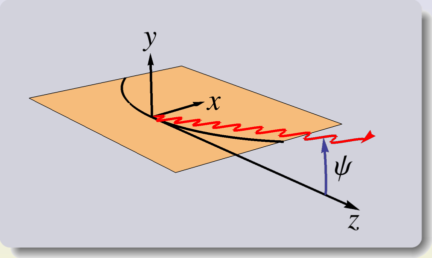
(the wake is in front of the particle where $s < 0$).

From Eq. (12.3) we see that the energy loss of *one electron* is proportional to N, which means that the total energy radiated by the beam is proportional to N^2. This is *coherent* radiation of the bunch. If it is not suppressed, it can exceed the *incoherent* radiation by orders of magnitude.

CSR wake and coherent synchrotron radiation

The longitudinal CSR impedance

$$Z_{csr}(\omega) = \frac{1}{c}\int_{-\infty}^{\infty} ds w_{csr}(s) e^{i\omega s/c} = \frac{Z_0 c}{4\pi}\frac{2}{3^{1/3}}\Gamma\left(\frac{2}{3}\right) e^{i\pi/6}\frac{(\omega/c)^{1/3}}{c\rho^{2/3}}.$$



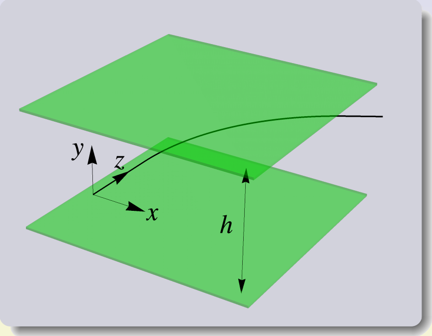
$\mathrm{Re}\, Z_{csr}$ is related to the energy loss, at frequency ω, of a point charge due to the synchrotron radiation (see Eq. (4.15)),

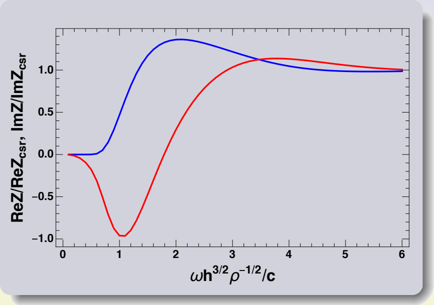
$$\frac{dP}{d\omega} = \frac{e^2}{\pi}\mathrm{Re}\, Z_{csr}(\omega)$$

One can find in textbooks on EM that this is the spectral power of synchrotron radiation in the range of frequencies $\omega \ll \omega_c$ with $\omega_c = \frac{3}{2}c\gamma^3/\rho$.

Shielding of CSR radiation

Introducing material boundaries close to the beam orbit suppresses the synchrotron radiation and the CSR wake. Shielded CSR impedance in the model of parallel plates has been studied in[38]; h is the full gap between the plates.



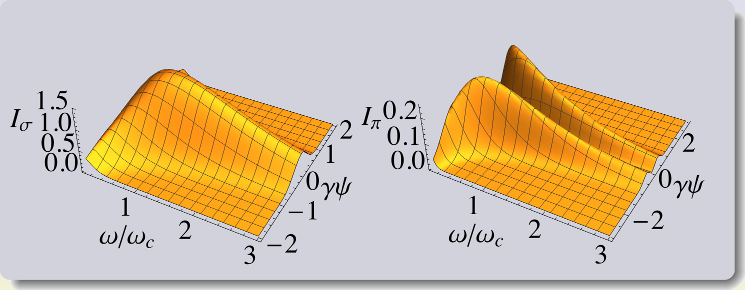


[38] J. B. Murphy, S. Krinsky, and R. L. Gluckstern. Longitudinal wakefield for an electron moving on a circular orbit. Part. Accel., **57** p. 9-64, 1997.

Explanation of the shielding effect

The angular spread of the synchrotron radiation at frequencies $\omega \ll \omega_c$ is of the order of

$$\psi \sim \left(\frac{c}{\omega\rho}\right)^{1/3}$$

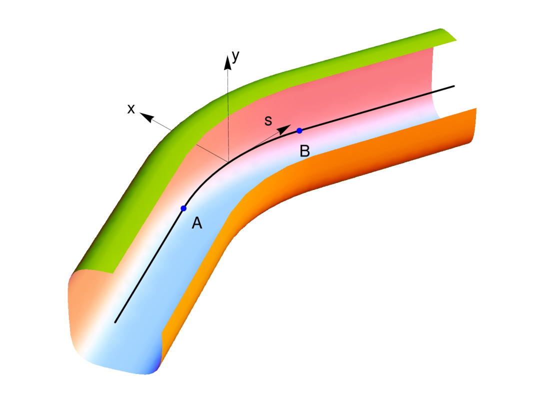


Hence, there are transverse wavenumbers in the radiation $k_\perp \sim k\psi$. If $h \lesssim 1/k_\perp$, these wavenumbers do not "fit" in the gap. The radiation is suppressed when

$$h \lesssim \frac{1}{k\psi} \sim \left(\frac{c}{\omega}\right)^{2/3} \rho^{1/3}$$

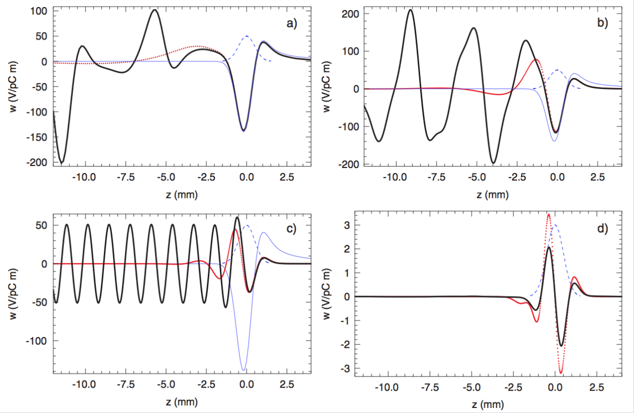
SCR wake in rectangular chambers

In this model it is assumed that the vacuum chamber has a rectangular $a \times b$ (no rounded corners). The beam trajectory consists of a straight line from $-\infty$, then a bend of a given radius ρ, then a straight line to $+\infty$.



SCR wake in rectangular chambers

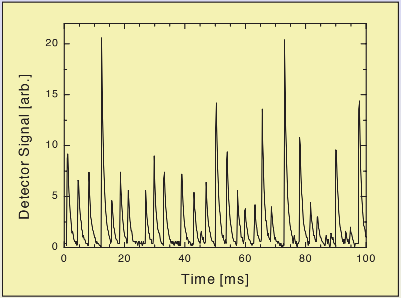
Examples of CSR wakes in a rectangular toroidal vacuum chamber[39]. Bunch length $\sigma_s = 0.5$ mm, $\rho = 1$ m, square cross-section of the vacuum chamber with the size 4 (a), 2 (b), 1 (c) and 0.5 (d) cm. The blue solid line shows the wake in free space, red—parallel plates model. Note the vertical scale.



[39] G. Stupakov and I. Kotelnikov. Phys. Rev. ST Accel. Beams 12, 104401 (2009)

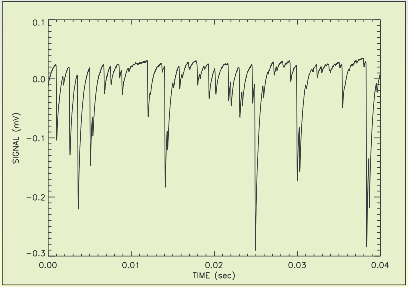
Bursts of coherent radiation

There have been many observations of bursts of radiation in light source storage rings, with $\lambda < \sigma_z$ (NSLS VUV, SURF at NIST, ALS at LBL, BESSY). Intensity $\propto N^2$, polarization of synchrotron radiation.



Bursts of CSR (far infrared) in NSLS VUV ring [Carr et al., NIM, 387, (2001)]. Frequency range from ~ 6 to ~ 60 GHz.

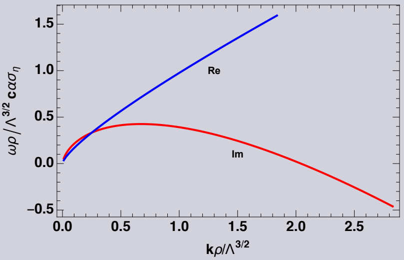
Bursts of CSR in BESSY II [G. Wustefeld et al.]. Typical wavelength ~ 0.5 mm.



CSR microwave instability

These bursts were explained by developing of the microwave instability due to the CSR wake. The simplest approach to explain the CSR driven instability is to use the Keil-Schnell theory[40]. The dispersion relation (10.13)

$$\frac{e^2}{m\gamma} Z_{csr}(ck) = \left(i \int_{-\infty}^{\infty} d\eta \frac{df_0/d\eta}{\omega + kc\alpha\eta} \right)^{-1}$$



The beam is unstable for such wavelengths that

$$k\rho < 2.0\Lambda^{3/2}.$$

$$\Lambda = \frac{1}{|\alpha|\gamma\sigma_\eta^2} \frac{I}{I_A} \frac{\rho}{\langle\rho\rangle}$$

The energy spread introduces Landau damping and stabilizes short wavelengths.

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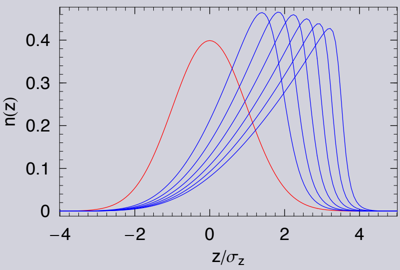
[40] Heifets and Stupakov, PRAB, **5**, 054402 (2002).

CSR equilibrium and Haïssinski equation

The CSR instability was also simulated with computer codes. The first step in these simulations is solving the Haïssinski equilibrium of the beam.



The solution depends on one dimensionless parameter $\mathcal{I}$. Haïssinski equilibria for , $\mathcal{I} = 3, \ldots, 8$

$$\mathcal{I} = \frac{N}{3^{1/3}\pi\gamma\nu_s\sigma_\eta}\frac{Cr_e}{\rho^{2/3}\sigma_z^{4/3}}$$

where C is the ring circumference.

CSR instability with account of shielding and equilibrium

Simulation of the threshold of the instability with shielding by two parallel plates[41]. Here $S = (3^{1/3}/4\pi)\mathcal{I}$.

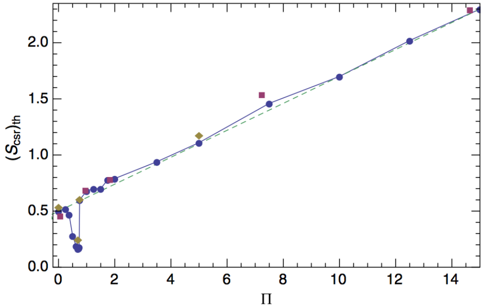


FIG. 3. For the CSR wake, threshold value of S_{csr} vs shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue circles), the LV code (red squares), and the VFP solver with twice stronger radiation damping (olive diamonds).

[41] K. L. F. Bane, Y. Cai, and G. Stupakov. PRST-AB 13, 104402 (2010).

CSR instability — experiment

The theoretical threshold of the instability is in good agreement with experimental observations at KARA light source in Germany[42]

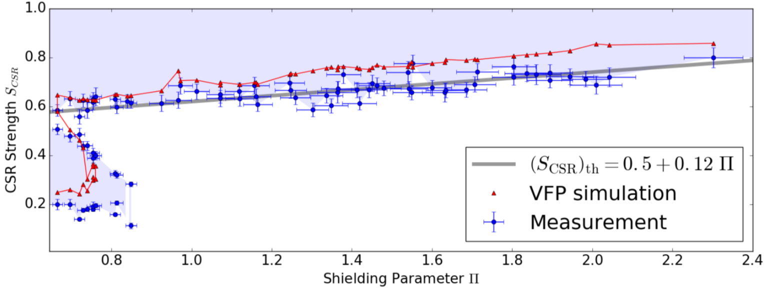
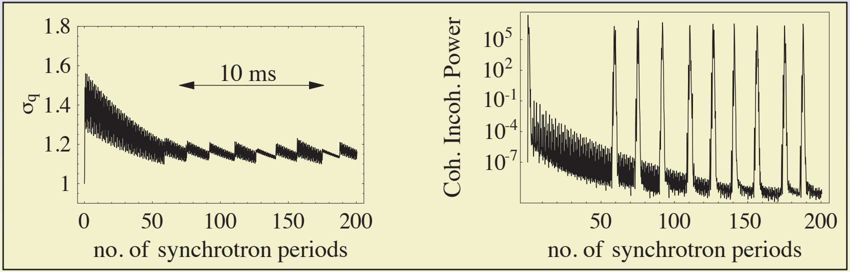


FIG. 5. CSR strength at the bursting thresholds as a function of the shielding parameter for measurements and VFP solver calculations for different machine settings. The measured area of instability is indicated as light blue area and confined by the measured thresholds (blue discs, already shown in Fig. 3) with the error bars displaying the standard deviation error of each measurement. The red triangles show the results from the VFP solver calculations at the corresponding machine settings (red line to guide the eye). The gray line indicates the linear scaling law for the main bursting threshold given by Eq. 3.

[42] M. Brosi et al. arXiv:1807.01145 (2018).

Computer simulations of nonlinear CSR instability

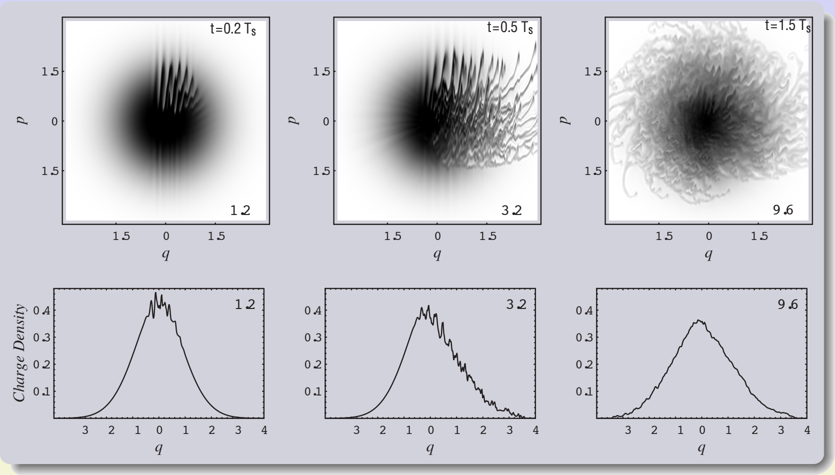
Computer simulations show what happens if the instability develops into nonlinear regime [43]. Numerical solution of Vlasov-Fokker-Planck equation, including CSR shielding with parallel plates and damping and quantum fluctuations due incoherent radiation.



Shown are normalized bunch length and ratio of coherent to incoherent power for NSLS VUV ring.

[43] Venturini and Warnock, PRL, 224802, 2002.

Nonlinear Regime—Computer Simulations



Courtesy of R. Warnock